

Efficient entanglement concentration for arbitrary less-entangled NOON state assisted with single photon

Lan Zhou¹, Yu-Bo Sheng,^{2,3}

¹ *College of Mathematics & Physics,
Nanjing University of Posts and Telecommunications,
Nanjing, 210003, China*

² *Institute of Signal Processing Transmission,
Nanjing University of Posts and Telecommunications,
Nanjing, 210003, China*

³ *Key Lab of Broadband Wireless Communication and Sensor Network Technology,
Nanjing University of Posts and Telecommunications,
Ministry of Education, Nanjing, 210003, China*

(Dated: September 11, 2012)

We put forward two efficient entanglement concentration protocols (ECPs) for distilling the maximally entangled NOON state from arbitrary less-entangled NOON state with only an auxiliary single photon. With the help of the weak cross-Kerr nonlinearities, both the two ECPs can be used repeatedly to get a high success probability. In the first ECP, the auxiliary single photon should be shared by the two parties say Alice and Bob. In the second ECP, the auxiliary single photon is only possessed by Bob, which can greatly increase the practical success probability by avoiding the transmission loss. Moreover, Bob can operate the whole protocol alone, which makes the protocol more simple. Therefore, our two ECPs, especially the second ECP may be more useful and convenient in the current quantum information processing.

PACS numbers: 03.67.Dd, 03.67.Hk, 03.65.Ud

I. INTRODUCTION

Entanglement plays a significant role in the quantum information mechanics, for it not only can hold the power for the quantum nonlocality, but also can provide wide applications in the quantum information processing (QIP) [1, 2]. For example, in the fields of quantum dense coding [3], quantum teleportation [4], quantum key distribution [5–7] and other quantum information processes [8–15], we need entangled quantum systems to setup the quantum channel. Recently, a special quantum state, which is so-called the NOON state has attracted great attention in the field of quantum information [16–23]. The NOON state can be written with the form

$$|NOON\rangle_{ab} = \frac{1}{\sqrt{2}}(|N, 0\rangle_{ab} + |0, N\rangle_{ab}), \quad (1)$$

which describes the state where all N particles are in the spatial mode a and none is in the spatial mode b , plus none in the mode a and all in the mode b . Due to its unique properties, the NOON state has provided many important applications. For example, it has been proved that the NOON state can serve as an important resource for Heisenberg-limited metrology and quantum lithography, where it is greatly sensitivity for optical interferometry and can approach the Heisenberg limit of $1/N$ [24–26]. Meanwhile, the NOON state also shows the de Broglie wavelength of λ/N for N -photon interference [21, 27–29]. It is worth noting that the power of NOON state lies in its entanglement. In all applications, the ideal NOON state should be the maximally entangled NOON state as Eq. (1). However, in practical application process, since the decoherence is unavoidable during the storage and transmission of particles over noisy channels, the quality of entanglement can be easily degraded. The noise may lead the maximally entangled NOON state degrade to the less-entangled state. The general form of the less-entangled NOON state can be written as

$$|NOON'\rangle_{ab} = \alpha|N, 0\rangle_{ab} + \beta|0, N\rangle_{ab}, \quad (2)$$

where α and β are the entanglement coefficients, $|\alpha|^2 + |\beta|^2 = 1$, and $\alpha \neq \beta$. In practical application, if the maximally entangled NOON state is polluted, we need to recover it into the maximally entangled NOON state.

The method for distilling the less-entangled pure state into the maximally entangled state is called the entanglement concentration [30]. In 1996, Bennett *et al.* proposed the first entanglement concentration protocol (ECP), which is known as the Schmidt projection method [30]. This protocol is regarded to be an exciting start for the study of the entanglement concentration. Since then, various interesting ECPs have been put forward, successively [31–47]. For example, In 1999, Bose *et al.* proposed an ECP based on entanglement swapping [31], which was later improved by Shi *et al.* [32]. In 2001, Zhao *et al.* and Yamamoto *et al.* proposed two similar concentration protocols based

on polarizing beam splitters (PBSs) independently [33, 34]. In 2010, the ECP for single-photon entanglement was proposed by Sheng *et al.* [36]. Recently, the ECPs for less-entangled W states were proposed [38, 45–47]. So far, most of the ECPs are focused on the two-particle entanglement, especially the encoding system in the polarization degree of freedom in the optics system. They cannot deal with the less-entangled NOON state. Recently, we have proposed an ECP for less-entangled NOON state [48]. However, this protocol is not optimal. In the protocol, two pairs of less-entangled NOON states are required, and after performing this ECP, at least one pair of maximally entangled NOON state can be obtained.

In the paper, we put forward two efficient ECPs for concentrating the less-entangled NOON state into the maximally entangled NOON state. Different from the ECP described in Ref. [48], our two ECPs only require one pair of less-entangled NOON state and a single auxiliary photon to complete the concentration task and can reach the same success probability as Ref. [48]. In the two ECPs, we adopt the weak cross-Kerr nonlinearity to construct the quantum nondemolition (QND) measurement, which makes both the two ECPs can be realized in current technology. Moreover, we will prove that both the two ECPs can be used repeatedly to get a high success probability. The first ECP is essentially inspired by the ECP in Ref. [36], as the single-photon entanglement is the simplest NOON state. The second ECP is more optimal than the first one, for it only requires the local single auxiliary photon, which can greatly avoid the photon loss during the transmission of the single auxiliary photon. Moreover, only Bob needs to perform the whole task which can simplify the practical experiment largely.

This paper is organized as follows: In Sec. II and Sec. III, we explain the basic principles of the first and the second ECP, respectively. In Sec. IV, we make a discussion and summary.

II. THE FIRST CONCENTRATION PROTOCOL FOR THE N-PHOTON NOON STATE

The cross-Kerr nonlinearity is the key element for both two ECPs. Therefore, before describing the first ECP, we would like to make a brief introduction on the cross-Kerr nonlinearity. The cross-Kerr nonlinearity has been widely used in the field of quantum information. It has played an important role in the construction of CNOT gate [49, 50], Bell-state analysis [51, 52], and so on [53–57]. Especially, the cross-Kerr nonlinearity is a powerful tool in the construction of the QND, which is widely used in the quantum entanglement concentration [35–38, 43–47]. The cross-Kerr nonlinearity can be described by its Hamiltonian as

$$H_{ck} = \hbar\chi\hat{n}_a\hat{n}_b, \quad (3)$$

where $\hbar\chi$ is the coupling strength of the nonlinearity, which depends on the cross-Kerr material [49, 50, 58]. \hat{n}_a and \hat{n}_b are the photon number operators for the spatial mode a and b. During the cross-Kerr interaction, a laser pulse in the coherent state $|\alpha\rangle$ interacts with photons through a proper cross-Kerr material. If the single photon is presented, it can induce a phase shift θ to the coherent state. The cross-Kerr interaction can be described as

$$U_{ck}|\psi\rangle|\alpha\rangle = (\gamma|0\rangle + \delta|1\rangle)|\alpha\rangle \rightarrow \gamma|0\rangle|\alpha\rangle + \delta|1\rangle|\alpha e^{i\theta}\rangle. \quad (4)$$

Here, $|0\rangle$ and $|1\rangle$ mean none photon and one photon, respectively. $\theta = \chi t$, and t means the interaction time for the signal with the nonlinear material. According to the Eq. (4), it can be easily found that the phase shift of the coherent state is directly proportional to the number of photons. Therefore, by measuring the phase shift of the coherent state, we can check the photon number without destroying the photons.

Now, we start to explain our first ECP for distilling the maximally entangled NOON state from arbitrary less-entangled NOON state. The principle of the ECP is shown in Fig. 1. We suppose that Alice (A) and Bob (B) share a less-entangled N-photon NOON state $|\phi\rangle_{a_1 b_1}$ in the spatial modes a_1 and b_1 , which is generated by the photon source S_1 . $|\phi\rangle_{a_1 b_1}$ can be written of the form

$$|\phi\rangle_{a_1 b_1} = \alpha|N, 0\rangle_{a_1 b_1} + \beta|0, N\rangle_{a_1 b_1}. \quad (5)$$

Here α and β are the entanglement coefficients of the NOON state, where $|\alpha|^2 + |\beta|^2 = 1$, and $\alpha \neq \beta$.

A single-photon entanglement source S_2 emits an auxiliary photon and sends it to Alice and Bob in the spatial modes a_2 and b_2 , which creates a single-photon entangled state of the form

$$|\phi\rangle_{a_2 b_2} = \alpha|1, 0\rangle_{a_2 b_2} + \beta|0, 1\rangle_{a_2 b_2}. \quad (6)$$

Here, for ensuring the coefficients of the auxiliary photon, we need to know the initial coefficients of the less-entangled NOON state, α and β . It is not difficult to know the values of the two coefficients by measuring an enough amount of the sample entangled NOON state [47].

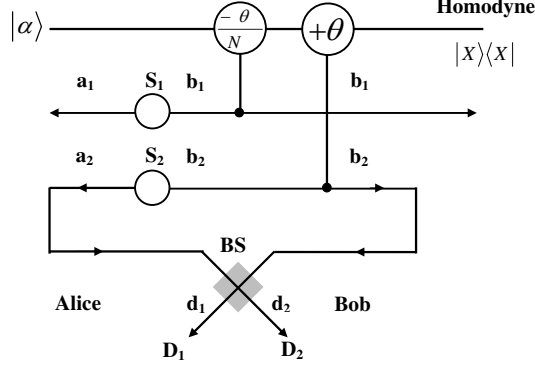


FIG. 1: A schematic diagram of our first ECP. In this protocol, an auxiliary photon which is shared by the two parties is used to concentrate the less-entangled NOON state. Bob makes the photons in the spatial mode b_1 and b_2 pass through two cross-Kerr nonlinearities, respectively, and selects the items $|N, 0, 0, 1\rangle_{a_1 b_1 a_2 b_2}$ and $|0, N, 1, 0\rangle_{a_1 b_1 a_2 b_2}$, which make the coherent state pick up $\pm\theta$. Then the two parties make the photons in the modes a_2 and b_2 pass through the 50:50 BS, and finally can obtain the maximally entangled NOON state. By making the discarded items pass through the BS, they can evolve to a new less-entangled NOON state and can be reconcentrated for the next round.

Then the whole $N+1$ photons system can be described as

$$\begin{aligned} |\Phi\rangle_{a_1 b_1 a_2 b_2} &= |\phi\rangle_{a_1 b_1} \otimes |\phi\rangle_{a_2 b_2} \\ &= (\alpha^2 |N, 0, 1, 0\rangle + \alpha\beta |N, 0, 0, 1\rangle + \alpha\beta |0, N, 1, 0\rangle + \beta^2 |0, N, 0, 1\rangle)_{a_1 b_1 a_2 b_2}. \end{aligned} \quad (7)$$

According to Fig. 1, Bob makes the photons in the spatial modes b_1 and b_2 pass through two cross-Kerr nonlinearities, respectively. Then, the $N+1$ photons system combined with the coherent state can evolve to

$$\begin{aligned} |\Phi\rangle_{a_1 b_1 a_2 b_2} \otimes |\alpha\rangle &\rightarrow \alpha^2 |N, 0, 1, 0\rangle_{a_1 b_1 a_2 b_2} |\alpha\rangle + \alpha\beta |N, 0, 0, 1\rangle_{a_1 b_1 a_2 b_2} |\alpha e^{i\theta}\rangle \\ &+ \alpha\beta |0, N, 1, 0\rangle_{a_1 b_1 a_2 b_2} |\alpha e^{-i\theta}\rangle + \beta^2 |0, N, 0, 1\rangle_{a_1 b_1 a_2 b_2} |\alpha\rangle. \end{aligned} \quad (8)$$

From Eq. (8), we can easily find that the items $|N, 0, 0, 1\rangle_{a_1 b_1 a_2 b_2}$ and $|0, N, 1, 0\rangle_{a_1 b_1 a_2 b_2}$ make the coherent state pick up the phase shifts of θ and $-\theta$, respectively, while other two items make the coherent state pick up no phase shift. As $\pm\theta$ is undistinguishable during the homodyne measurement, Bob selects the items corresponding to the phase shift of $\pm\theta$, and discards the other items. Then the Eq. (7) can collapse to

$$|\Phi_1\rangle_{a_1 b_1 a_2 b_2} = \frac{1}{\sqrt{2}}(|N, 0, 0, 1\rangle_{a_1 b_1 a_2 b_2} + |0, N, 1, 0\rangle_{a_1 b_1 a_2 b_2}), \quad (9)$$

with the success probability of $2|\alpha\beta|^2$.

Then Alice and Bob make the photons in the modes a_2 and b_2 pass through the 50:50 BS and take the Bell measurement, which makes

$$\begin{aligned} \hat{a}_2^\dagger|0\rangle &= \frac{1}{\sqrt{2}}(\hat{d}_1^\dagger|0\rangle - \hat{d}_2^\dagger|0\rangle) \\ \hat{b}_2^\dagger|0\rangle &= \frac{1}{\sqrt{2}}(\hat{d}_1^\dagger|0\rangle + \hat{d}_2^\dagger|0\rangle). \end{aligned} \quad (10)$$

Here, a_j , b_j and d_j ($j = 1, 2$) are the creation operators for the spatial mode a_j , b_j and d_j , respectively. The creation operators obey the rules that $\hat{i}_j^\dagger|0\rangle = |1\rangle_{i_j}$ and $(\hat{i}_j^\dagger)^N|0\rangle = \sqrt{N}|N\rangle_{i_j}$, where $i = a, b, d$ and $j = 1, 2$. After the BS, Eq. (9) can ultimately evolve to

$$|\Phi_1\rangle_{a_1 b_1 d_1 d_2} = \frac{1}{\sqrt{2}}(|N, 0, 1, 0\rangle + |0, N, 1, 0\rangle)_{a_1 b_1 d_1 d_2} + \frac{1}{\sqrt{2}}(|N, 0, 0, 1\rangle - |0, N, 0, 1\rangle)_{a_1 b_1 d_1 d_2}. \quad (11)$$

If the detector D_1 fires, Eq. (11) will collapse to

$$|\phi_1\rangle_{a_1 b_1} = \frac{1}{\sqrt{2}}(|N, 0\rangle_{a_1 b_1} + |0, N\rangle_{a_1 b_1}), \quad (12)$$

while if the detector D_2 fires, Eq. (11) will collapse to

$$|\phi'_1\rangle_{a_1 b_1} = \frac{1}{\sqrt{2}}(|N, 0\rangle_{a_1 b_1} - |0, N\rangle_{a_1 b_1}). \quad (13)$$

Both Eq. (12) and Eq. (13) are the maximally entangled N -photon NOON states and there is only a phase difference between them. Eq. (13) can be converted to Eq. (12) by the phase flip operation with the help of a half-wave plate. So far, our first concentration protocol is completed. In the protocol, with the help of the weak cross-Kerr nonlinearity and the BS, by selecting the items which make the coherent state pick up the phase shift $\pm\theta$, Alice and Bob can distill the maximally entangled N -photon NOON state with the success probability $P=2|\alpha\beta|^2$.

Moreover, we will prove that the ECP can be reused to further concentrate the discarded items. The discarded items which make the coherent state pick up no phase shift can be written as

$$|\Phi_2\rangle_{a_1 b_1 a_2 b_2} = \alpha^2 |N, 0, 1, 0\rangle_{a_1 b_1 a_2 b_2} + \beta^2 |0, N, 0, 1\rangle_{a_1 b_1 a_2 b_2}. \quad (14)$$

Alice and Bob also make the photons in the spatial modes a_2 and b_2 pass through the BS, and they can finally get

$$|\Phi_2\rangle_{a_1 b_1 d_1 d_2} = (\alpha^2 |N, 0, 1, 0\rangle + \beta^2 |0, N, 1, 0\rangle)_{a_1 b_1 d_1 d_2} + (\alpha^2 |N, 0, 0, 1\rangle - \beta^2 |0, N, 0, 1\rangle)_{a_1 b_1 d_1 d_2}. \quad (15)$$

It is obvious if the detector D_1 fires, Eq. (15) will collapse to

$$|\phi_2\rangle_{a_1 b_1} = \alpha^2 |N, 0\rangle_{a_1 b_1} + \beta^2 |0, N\rangle_{a_1 b_1}, \quad (16)$$

while if the detector D_2 fires, Eq. (15) will collapse to

$$|\phi'_2\rangle_{a_1 b_1} = \alpha^2 |N, 0\rangle_{a_1 b_1} - \beta^2 |0, N\rangle_{a_1 b_1}. \quad (17)$$

Similar with Eq. (12) and Eq. (13), Eq. (17) can be easily converted to Eq. (16) by passing it through a half-wave plate. We can find that Eq. (16) has the similar form with Eq. (5), that is to say, Eq. (16) is a new less-entangled N -photon NOON state and can be reconcentrated for the next round. In the second concentration round, the single photon source S_2 emits an auxiliary photon with the form

$$|\phi'\rangle_{a_2 b_2} = \alpha^2 |1, 0\rangle_{a_2 b_2} + \beta^2 |0, 1\rangle_{a_2 b_2}. \quad (18)$$

Following the steps in the first concentration round, Bob makes the photons in the spatial modes b_1 and b_2 pass through the QND, and the whole $N+1$ photon system combined with the coherent state can be described as

$$\begin{aligned} |\Phi'\rangle_{a_1 b_1 a_2 b_2} \otimes |\alpha\rangle &= |\phi_2\rangle_{a_1 b_1} \otimes |\phi'\rangle_{a_2 b_2} \otimes |\alpha\rangle \\ &\rightarrow \alpha^4 |N, 0, 1, 0\rangle_{a_1 b_1 a_2 b_2} |\alpha\rangle + \alpha^2 \beta^2 |N, 0, 0, 1\rangle_{a_1 b_1 a_2 b_2} |\alpha e^{i\theta}\rangle \\ &\quad + \alpha^2 \beta^2 |0, N, 1, 0\rangle_{a_1 b_1 a_2 b_2} |\alpha e^{-i\theta}\rangle + \beta^4 |0, N, 0, 1\rangle_{a_1 b_1 a_2 b_2} |\alpha\rangle. \end{aligned} \quad (19)$$

Then Bob still selects the items which make the coherent state pick up the phase shift $\pm\theta$, and the Eq. (19) can collapse to Eq. (9). For obtaining the maximally entangled N -photon NOON state, Alice and Bob also make the photons in the spatial mode a_2 and b_2 pass through the BS. After the BS, Eq. (9) can evolve to Eq. (12) and Eq. (13), which are the maximally entangled N -photon NOON states. Therefore, in the second concentration round, Eq. (16) can be successfully converted to the maximally entangled N -photon NOON state with the probability $P_2 = \frac{2|\alpha\beta|^4}{|\alpha|^4 + |\beta|^4}$, where the subscript '2' means in the second concentration round.

Similarly, by making the discarded items in the second concentration round pass through the BS, it can ultimately evolve to

$$|\phi_3\rangle_{a_1 b_1} = \alpha^4 |N, 0\rangle_{a_1 b_1} + \beta^4 |0, N\rangle_{a_1 b_1}, \quad (20)$$

which is a new less-entangled state and can be reconcentrated in the third round. Therefore, in this way, our ECP can be used repeatedly to further concentrate less-entangled NOON state.

III. THE SECOND ECP OF THE N -PHOTONS NOON STATE

The schematic diagram of our second ECP is shown in Fig. 2. We also suppose that Alice and Bob share a less-entangled N -photon NOON state $|\phi\rangle_{a_1 b_1}$ with the form of Eq. (5), which is generated by S_1 . Here, single-photon source S_2 emits an auxiliary photon and only sends it to Bob in the spatial mode b_2 . Bob makes this auxiliary photon

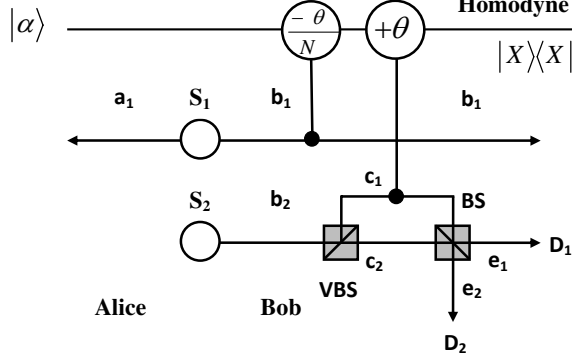


FIG. 2: A schematic diagram of our second ECP. The single photon source S_2 emits an auxiliary photon and only sends it to Bob. By passing it through the VBS, a single-photon entangled state can be created. Bob makes the photons in the mode b_1 and c_1 pass through two cross-Kerr nonlinearities, and selects the items which make the coherent state pick up the phase shift of $\pm\theta$. Then Bob makes the photons in the modes c_1 and c_2 pass through the BS and takes the Bell measurement. If a suitable VBS with the transmission of $t = \alpha^2$ can be provided, they can ultimately obtain the maximally entangled NOON state. Moreover, by choosing suitable VBS with the transmission of $t_K = \frac{|\alpha|^{2K}}{|\alpha|^{2K} + |\beta|^{2K}}$, the ECP can be used repeatedly to get a high success probability.

pass through a variable beam splitter (VBS) with the transmission of t , which can create an single-photon entangled state between the spatial modes c_1 and c_2 with the form

$$|\psi\rangle_{c_1 c_2} = \sqrt{1-t}|1, 0\rangle_{c_1 c_2} + \sqrt{t}|0, 1\rangle_{c_1 c_2}. \quad (21)$$

Then the whole $N+1$ photons system can be described as

$$\begin{aligned} |\psi\rangle_{a_1 b_1 c_1 c_2} &= |\phi\rangle_{a_1 b_1} \otimes |\psi\rangle_{c_1 c_2} \\ &= \alpha\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2} + \alpha\sqrt{t}|N, 0, 0, 1\rangle_{a_1 b_1 c_1 c_2} \\ &\quad + \beta\sqrt{1-t}|0, N, 1, 0\rangle_{a_1 b_1 c_1 c_2} + \beta\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2}. \end{aligned} \quad (22)$$

Bob makes the photons in the spatial mode b_1 and c_1 pass through the QND, and the whole $N+1$ photon system combined with the coherent state can evolve to

$$\begin{aligned} |\Psi\rangle_{a_1 b_1 c_1 c_2} &= |\psi\rangle_{a_1 b_1 c_1 c_2} \otimes |\alpha\rangle \\ &\rightarrow \alpha\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2} |\alpha e^{i\theta}\rangle + \alpha\sqrt{t}|N, 0, 0, 1\rangle_{a_1 b_1 c_1 c_2} |\alpha\rangle \\ &\quad + \beta\sqrt{1-t}|0, N, 1, 0\rangle_{a_1 b_1 c_1 c_2} |\alpha\rangle + \beta\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2} |\alpha e^{-i\theta}\rangle. \end{aligned} \quad (23)$$

It can be seen that the items $\alpha\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2}$ and $\beta\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2}$ can make the coherent state pick up the phase shift of $\pm\theta$, while both the items $\alpha\sqrt{t}|N, 0, 0, 1\rangle_{a_1 b_1 c_1 c_2}$ and $\beta\sqrt{1-t}|0, N, 1, 0\rangle_{a_1 b_1 c_1 c_2}$ make it pick up no phase shift. Bob selects the items corresponding to the phase shift $\pm\theta$ and discards other items. Then Eq. (22) can collapse to

$$|\Psi_1\rangle_{a_1 b_1 c_1 c_2} = \alpha\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2} + \beta\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2},$$

with the success probability of $2|\alpha\beta|^2$.

Then, Bob makes the photons in the spatial modes c_1 and c_2 pass through the 50:50 BS, which makes

$$\begin{aligned} \hat{c}_1^\dagger|0\rangle &= \frac{1}{\sqrt{2}}(\hat{e}_1^\dagger|0\rangle + \hat{e}_2^\dagger|0\rangle) \\ \hat{c}_2^\dagger|0\rangle &= \frac{1}{\sqrt{2}}(\hat{e}_1^\dagger|0\rangle - \hat{e}_2^\dagger|0\rangle). \end{aligned} \quad (24)$$

After the BS, Eq. (24) can evolve to

$$\begin{aligned} |\Psi_1\rangle_{a_1 b_1 e_1 e_2} &= \frac{\alpha\sqrt{1-t}}{\sqrt{2}}|N, 0, 1, 0\rangle_{a_1 b_1 e_1 e_2} + \frac{\beta\sqrt{t}}{\sqrt{2}}|0, N, 1, 0\rangle_{a_1 b_1 e_1 e_2} \\ &\quad + \frac{\alpha\sqrt{1-t}}{\sqrt{2}}|N, 0, 0, 1\rangle_{a_1 b_1 e_1 e_2} - \frac{\beta\sqrt{t}}{\sqrt{2}}|0, N, 0, 1\rangle_{a_1 b_1 e_1 e_2}. \end{aligned} \quad (25)$$

It is obvious that if the detector D_1 fires, Eq. (25) will collapse to

$$|\psi_1\rangle_{a_1 b_1} = \alpha\sqrt{1-t}|N, 0\rangle_{a_1 b_1} + \beta\sqrt{t}|0, N\rangle_{a_1 b_1}, \quad (26)$$

while if the detector D_2 fires, Eq. (25) will evolve to

$$|\psi_2\rangle_{a_1 b_1} = \alpha\sqrt{1-t}|N, 0\rangle_{a_1 b_1} - \beta\sqrt{t}|0, N\rangle_{a_1 b_1}. \quad (27)$$

If Bob gets Eq. (27), he can convert it to Eq. (26) by passing it through the half-wave plate. According to Eq. (26), if we can find a suitable VBS with the transmission $t = \alpha^2$, Eq. (26) can evolve to Eq. (12), which is the maximally entangled NOON state. So far, our second ECP is completed. With the help of the weak cross-Kerr nonlinearity and the VBS, we can ultimately distill the maximally entangled N-photon NOON state with the probability of $P=2|\alpha\beta|^2$.

Moreover, we can also prove that the second ECP can be used repeatedly. When $t = \alpha^2$, the discarded items in the first concentration round can be written as

$$|\Psi_2\rangle_{a_1 b_1 c_1 c_2} = \alpha^2|N, 0, 0, 1\rangle_{a_1 b_1 c_1 c_2} + \beta^2|0, N, 1, 0\rangle_{a_1 b_1 c_1 c_2}. \quad (28)$$

Bob still makes the photons in the spatial modes c_1 and c_2 pass through the BS, and Eq. (28) can evolve to

$$\begin{aligned} |\Psi_2\rangle_{a_1 b_1 e_1 e_2} &= \frac{\alpha^2}{\sqrt{2}}|N, 0, 1, 0\rangle_{a_1 b_1 e_1 e_2} + \frac{\beta^2}{\sqrt{2}}|0, N, 1, 0\rangle_{a_1 b_1 e_1 e_2} \\ &+ \frac{\alpha^2}{\sqrt{2}}|N, 0, 0, 1\rangle_{a_1 b_1 e_1 e_2} - \frac{\beta^2}{\sqrt{2}}|0, N, 0, 1\rangle_{a_1 b_1 e_1 e_2}. \end{aligned} \quad (29)$$

Therefore, if the detector D_1 fires, Eq. (29) will collapse to Eq. (16), while if the detector D_2 fires, Eq. (29) will collapse to Eq. (17). Eq. (17) can be converted to Eq. (16) by the phase flip operation. According to the description in Sec. II, Eq. (16) is the new less entangled N-photon NOON states and can be reconcentrated in the next round.

In the second concentration round, the single photon source S_2 emits another auxiliary photon and sends it to Bob. By passing it through the VBS, a new single-photon entangled state with the form of Eq. (21) can be created. Bob still makes the photons in the spatial modes b_1 and c_1 pass through the QND. Then the whole $N+1$ photons system combined with the coherent state can be written as

$$\begin{aligned} |\psi'\rangle_{a_1 b_1 c_1 c_2} &= |\phi_2\rangle_{a_1 b_1} \otimes |\psi\rangle_{c_1 c_2} \otimes |\alpha\rangle \\ &\rightarrow \alpha^2\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2}|\alpha e^{i\theta}\rangle + \alpha^2\sqrt{t}|N, 0, 0, 1\rangle_{a_1 b_1 c_1 c_2}|\alpha\rangle \\ &+ \beta^2\sqrt{1-t}|0, N, 1, 0\rangle_{a_1 b_1 c_1 c_2}|\alpha\rangle + \beta^2\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2}|\alpha e^{-i\theta}\rangle. \end{aligned} \quad (30)$$

Bob still selects the items which make the coherent state pick up the phase shift $\pm\theta$, then Eq. (30) can evolve to

$$|\Psi'_1\rangle_{a_1 b_1 c_1 c_2} = \alpha^2\sqrt{1-t}|N, 0, 1, 0\rangle_{a_1 b_1 c_1 c_2} + \beta^2\sqrt{t}|0, N, 0, 1\rangle_{a_1 b_1 c_1 c_2}, \quad (31)$$

with the success probability of $\frac{2|\alpha\beta|^4}{|\alpha|^4+|\beta|^4}$.

Next, Bob makes the photons in the spatial modes c_1 and c_2 pass through the BS, and the Eq. (31) can finally evolve to

$$|\psi'_1\rangle_{a_1 b_1} = \alpha^2\sqrt{1-t}|N, 0\rangle_{a_1 b_1} + \beta^2\sqrt{t}|0, N\rangle_{a_1 b_1}. \quad (32)$$

If we can find a suitable VBS with the transmission $t_2 = \frac{|\alpha|^4}{|\alpha|^4+|\beta|^4}$, where the subscript '2' means in the second concentration round, Eq. (32) can evolve to Eq. (12). That is to say, Eq. (16) can be ultimately recovered to the maximally entangled NOON state with the probability $P_2 = \frac{2|\alpha\beta|^4}{|\alpha|^4+|\beta|^4}$. Similarly, by passing the discarded items in the second concentration round through the BS, the discarded items can also evolve to Eq. (20), which can be reconcentrated in the next round. Therefore, by choosing the suitable VBS with the transmission $t_K = \frac{|\alpha|^{2^K}}{|\alpha|^{2^K}+|\beta|^{2^K}}$, where the subscript 'K' means the iteration time, the second ECP can be used repeatedly to distill the maximally entangled NOON state.

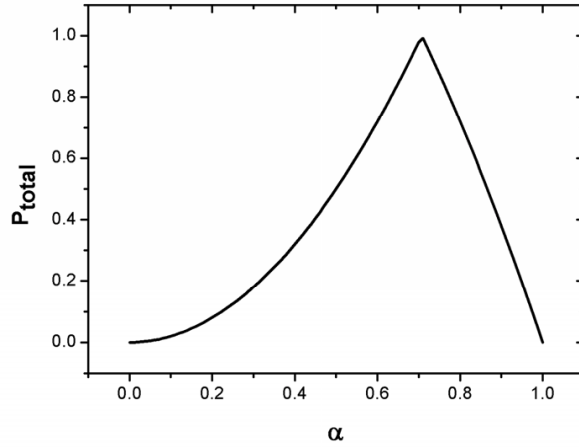


FIG. 3: The success probability (P_{total}) for obtaining a maximally entangled N-photon NOON state after the concentration protocol being operated for K times. For numerical simulation, we choose $K = 10$. It can be seen that the value of P_{total} largely depends on the initial coefficient α . When $\alpha = \frac{1}{\sqrt{2}}$, P_{total} reaches the maximum as 1.

Finally, we would like to calculate the total success probability of these two ECPs. Both the two ECPs have the same success probability. In each concentration round, the success probability can be shown as

$$\begin{aligned}
 P_1 &= 2|\alpha\beta|^2 \\
 P_2 &= \frac{2|\alpha\beta|^4}{|\alpha|^4 + |\beta|^4} \\
 P_3 &= \frac{2|\alpha\beta|^8}{(|\alpha|^4 + |\beta|^4)(|\alpha|^8 + |\beta|^8)} \\
 P_4 &= \frac{2|\alpha\beta|^{16}}{(|\alpha|^4 + |\beta|^4)(|\alpha|^8 + |\beta|^8)(|\alpha|^{16} + |\beta|^{16})} \\
 &\dots\dots \\
 P_K &= \frac{2|\alpha\beta|^{2^K}}{(|\alpha|^4 + |\beta|^4)(|\alpha|^8 + |\beta|^8) \dots (|\alpha|^{2^K} + |\beta|^{2^K})}.
 \end{aligned} \tag{33}$$

$$\tag{34}$$

In theory, the ECPs can be reused indefinitely, the total success probability for distilling the maximally entangled N-photon NOON state from the less-entangled NOON state equals the sum of the probability in each concentration round.

$$P_{total} = P_1 + P_2 + \dots + P_K = \sum_{K=1}^{\infty} P_K \tag{35}$$

It can be found that if the initial entangled state is the maximally entangled NOON state, where $\alpha = \beta = \frac{1}{\sqrt{2}}$, the probability $P_{total} = \lim(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K} + \dots) = 1$, while if $\alpha \neq \beta$, the $P_{total} < 1$. Here, we choose $K = 10$ as a proper approximation and calculate the value of P_{total} in different initial entanglement coefficient. Fig. 3 shows the value of P_{total} as a function of the entanglement coefficient α . It can be found that P_{total} largely depends on the initial entanglement state. The higher initial entanglement can lead to the greater P_{total} .

IV. DISCUSSION AND SUMMARY

In the paper, we presented two efficient ECPs for distilling the maximally entangled N-photon NOON state from arbitrary less-entangled NOON state. In our ECPs, only a pair of less-entangled NOON state and an auxiliary single photon are required to complete the task. It is interesting to compare these two ECPs with other ones. In most

conventional ECPs, they require two similar copies of less-entangled states to obtain one pair of maximally entangled state. Our two ECPs only require one pair of less-entangled state, and can reach the same high success probability as the convention ECPs. With the help of cross-Kerr nonlinearity, after performing these ECPs, both the maximally entangled NOON state and the less-entangled NOON state can be remained for various applications. During these ECPs, they only require the weak cross-Kerr nonlinearity, which can greatly reduce the experimental difficulties. As pointed by Refs. [49, 50], in a practical operation, we should require $\alpha\theta > 1$ to make the phase shift reach an observed value. So in the condition of weak cross-Kerr nonlinearity (θ is small), the requirement can be satisfied by large amplitude of the coherent state. Certainly, both ECPs require to know the initial coefficients α and β in advance. Meanwhile, they also should make the coherent state $|\alpha\rangle$ pick up different phase shifts, with one is $-\frac{\theta}{N}$ and the other is θ . According to the relationship of $\theta = \chi t$, it can be realized by selecting the different Kerr materials or controlling the coupling time t .

In the first ECP, the auxiliary photon should be shared by Alice and Bob in distant locations. Moreover, after passing through the QND, the auxiliary photon should be sent back and will be transmitted in a second time. However, as pointed out by Ref. [59], the photon loss is unavoidable during the transmission process due to the noisy environment. It will greatly limit this ECP in the practical application in quantum information processing. Fortunately, in the second ECP, the auxiliary photon entanglement can be created locally and only possessed by Bob, which makes this ECP more powerful than the first one. Moreover, only Bob needs to operate this concentrate step, which leads this ECP more simple to be realized. In the second ECP, the VBS is the key element for creating the local single-photon entanglement. According to the initial coefficients of the less-entangled NOON state, they should adjust the different transmission t to satisfy $t_K = \frac{|\alpha|^{2K}}{|\alpha|^{2K} + |\beta|^{2K}}$ in the K th concentration round. Certainly, in the first ECP, the single-photon entanglement created by S_2 can also be reviewed as the single-photon source plus the VBS. The VBS is a common optical element in current technology. Recently, using the VBS, Osorio *et al.* reported their experimental results about the heralded photon amplification for quantum communication [60]. They adjusted the transmission of VBS to increase the probability of the single photon $|1\rangle$ from a mixed state $\eta_t|1\rangle\langle 1| + (1 - \eta_t)|0\rangle\langle 0|$. In their protocol, they adjust the splitting ratio of VBS from 50:50 to 90:10 to increase the visibility from $46.7 \pm 3.1\%$ to $96.3 \pm 3.8\%$.

In summary, we put forward two efficient ECPs for distilling the maximally entangled N -photon NOON state from arbitrary less-entangled NOON state. In both two ECPs, we only require a pair of less-entangled NOON state and an auxiliary single photon to complete the task. These two ECPs have two advantages: First, they do not require the large cross-Kerr nonlinearity, and the weak cross-Kerr nonlinearity can achieve the task. Second, with the help of the weak cross-Kerr nonlinearities, both two ECPs can be used repeatedly and have the same high success probability. Moreover, the second ECP is more optimal than the first one for two main reasons. First, all the operations in the second ECP are local, while those in the first ECP are nonlocal. Second, in the second protocol, benefit from the VBS, the requirement for the single-photon source is much lower. The two advantages can simply the operation and make the second ECP can be realized much easier under present experimental conditions. Therefore, our two ECPs, especially the second ECP may be useful and convenient in the current quantum information processing.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China under Grant No. 11104159, Open Research Fund Program of the State Key Laboratory of Low-Dimensional Quantum Physics Scientific, Tsinghua University, Open Research Fund Program of National Laboratory of Solid State Microstructures under Grant No. M25020 and M25022, Nanjing University, Scientific Research Foundation of Nanjing University of Posts and Telecommunications under Grant No. NY211008, and the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

-
- [1] Nielsen M.A., Chuang I.L.: Quantum computation and quantum information (Cambridge University Press, Cambridge, England, 2000).
 - [2] Gisin N., Ribordy G., Tittel W., Zbinden H.: Quantum cryptography. *Rev. Mod. Phys.* **74**, 145 (2002).
 - [3] Bennett C.H., Wiesner S.J.: Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. *Phys. Rev. Lett.* **69**, 2881 (1992).
 - [4] Bennett C.H., Brassard G., Crepeau C., Jozsa R., Peres A., Wootters W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **70**, 1895 (1993).
 - [5] Ekert A.K.: Quantum cryptography based on Bell's theorem. *Phys. Rev. Lett.* **67**, 661 (1991).

- [6] Deng F.G., Long G.L.: Controlled order rearrangement encryption for quantum key distribution. *Phys. Rev. A* **68**, 042315 (2003).
- [7] Li X.H., Deng F.G., Zhou H.Y.: Efficient quantum key distribution over a collective noise channel. *Phys. Rev. A* **78**, 022321 (2008).
- [8] Long G.L., Liu X.S.: Theoretically efficient high-capacity quantum-key-distribution scheme. *Phys. Rev. A*, **65**, 032302 (2002).
- [9] Deng F.G., Long G.L., Liu X.S.: Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block. *Phys. Rev. A*, **68**, 042317 (2003).
- [10] Wang C., Deng F.G., Li Y.S., Liu X.S., Long G.L.: Quantum secure direct communication with high-dimension quantum superdense coding. *Phys. Rev. A* **71**, 044305 (2005).
- [11] Hillery M., Bužek V., Berthiaume A.: Quantum secret sharing. *Phys. Rev. A* **59**, 1829 (1999).
- [12] Karlsson A., Koashi M., Imoto N.: Quantum entanglement for secret sharing and secret splitting. *Phys. Rev. A* **59**, 162 (1999).
- [13] Xiao L., Long G.L., Deng F.G., Pan J.W.: Efficient multiparty quantum-secret-sharing schemes. *Phys. Rev. A* **69**, 052307 (2004).
- [14] Divincenzo D.P.: Quantum computation. *Science* **270**, 255 (1995).
- [15] Bennett C.H., Divincenzo D.P.: Quantum information and computation. *Nature (London)* **404**, 247 (2000).
- [16] Dowling J.P.: Quantum optical metrology-the lowdown on high-NOON states. *Contemp. Phys.* **49**, 125 (2008).
- [17] Huelga S.F., Macchiavello C., Pellizzari T., Ekert A.K.: Improvement of frequency standards with quantum entanglement. *Phys. Rev. Lett.* **79**, 3865 (1997).
- [18] Resch K.J., Pagnell K.L., Prevedel R., Gilchrist A., Pryde G.J., Ó'Brien J.L., White A.G.: Time-reversal and super-resolving phase measurements. *Phys. Rev. Lett.* **98**, 223601 (2007).
- [19] Chen Y.A., Bao X.H., Yuan Z.S., Chen S., Zhao B., Pan J.W.: Heralded generation of an atomic NOON state. *Phys. Rev. Lett.* **104**, 043601 (2010).
- [20] Mitchell M.W., Lundeen J.S., Steinberg A.M.: Super-resolving phase measurements with a multiphoton entangled state. *Nature*, **429**, 161 (2004).
- [21] Walther P., Pan J.W., Aspelmeyer M., Ursin R., Gasparoni S., Zeilinger A.: De Broglie wavelength of a non-local four-photon state. *Nature*, **429**, 158 (2004).
- [22] Nagata T., Okamoto R., Ó'Brien J.L., Sasaki K., Takeuchi S.: Beating the standard quantum limit with four-entangled photons. *Science*, **316**, 726 (2007).
- [23] Israel Y., Afek I., Rosen S., Ambar O., Silberberg Y.: Experimental tomography of NOON states with large photon numbers. *Phys. Rev. A* **85**, 022115 (2012).
- [24] Boto A.N., Kok P., Abrams D.S., Braunstein S.L., Williams C.P., Dowling J.P.: Quantum interferometric optical lithography: exploiting entanglement to beat the diffraction limit. *Phys. Rev. Lett.* **85**, 2733, (2000).
- [25] Bollinger J.J., Itano W.M., Wineland D.J., Heinzen D.J.: Optimal frequency measurements with maximally correlated states. *Phys. Rev. A* **54**, R4649 (1996).
- [26] Eckert K., Hyllus P., Bruss D., et al.: Differential atom interferometry beyond the standard quantum limit. *Phys. Rev. A* **73**, 013814 (2006).
- [27] D'Angelo M., Chekhova M.V., Shih Y.: Two-photon diffraction and quantum lithography. *Phys. Rev. Lett.* **87**, 013602 (2001).
- [28] Sun F.W., Ou Z.Y., Guo G.C.: Simple proof of fault tolerance in the graph-state model. *Phys. Rev. A* **73**, 032308 (2006).
- [29] Liu B., Ou Z.Y.: Feasibility of Bell tests with the W state. *Phys. Rev. A* **81**, 033823 (2010).
- [30] Bennett C.H., Bernstein H.J., Popescu S., Schumacher B.: Concentrating partial entanglement by local operations. *Phys. Rev. A* **53**, 2046 (1996).
- [31] Bose S., Vedral V., Knight P.L.: Purification via entanglement swapping and conserved entanglement. *Phys. Rev. A* **60**, 194 (1999).
- [32] Shi B.S., Jiang Y.K., Guo G.C.: Optimal entanglement purification via entanglement swapping. *Phys. Rev. A* **62**, 054301 (2000).
- [33] Zhao Z., Pan J.W., Zhan M.S.: Practical scheme for entanglement concentration. *Phys. Rev. A* **64**, 014301 (2001).
- [34] Yamamoto T., Koashi M., Imoto N.: Concentration and purification scheme for two partially entangled photon pairs. *Phys. Rev. A* **64**, 012304 (2001).
- [35] Sheng Y.B., Deng F.G., Zhou H.Y.: Nonlocal entanglement concentration scheme for partially entangled multipartite systems with nonlinear optics. *Phys. Rev. A* **77**, 062325 (2008).
- [36] Sheng Y.B., Deng F.G., Zhou H.Y.: Single-photon entanglement concentration for long distance quantum communication. *Quant. Inf. & Comput.* **10**, 272 (2010).
- [37] Sheng Y.B., Zhou L., Zhao S.M., Zheng B.Y.: Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs. *Phys. Rev. A* **85**, 012307 (2012).
- [38] Sheng Y.B., Zhou L., Zhao S.M.: Efficient two-step entanglement concentration for arbitrary W states. *Phys. Rev. A* **85**, 044305 (2012).
- [39] Sheng Y.B., Deng F.G., Zhou H.Y.: Efficient polarization entanglement concentration for electrons with charge detection. *Phys. Lett. A* **373**, 1823 (2009).
- [40] Paunković N., Omar Y., Bose S., Vedral V.: Entanglement concentration using quantum statistics. *Phys. Rev. Lett.* **88**, 187903 (2002).

- [41] Wang X.B., Fan H.: Entanglement concentration by ordinary linear optical devices without postselection. *Phys. Rev. A* **68**, 060302 (2003).
- [42] Wang C., Zhang Y., Jin G.S.: Entanglement purification and concentration of electron-spin entangled states using quantum-dot spins in optical microcavities. *Phys. Rev. A* **84**, 032307 (2011).
- [43] Deng F.G.: Optimal nonlocal multipartite entanglement concentration based on projection measurements. *Phys. Rev. A* **85**, 022311 (2012).
- [44] Xiong. W., Ye L.: Schemes for entanglement concentration of two unknown partially entangled states with cross-Kerr nonlinearity. *J. Opt. Soc. Am. B* **28**, 2030 (2011).
- [45] Wang H.F., Zhang S., Yeon K.H.: Linear-optics-based entanglement concentration of unknown partially entangled three photon W states. *J. Opt. Soc. Am. B* **27**, 2159 (2010).
- [46] Gu B.: Single-photon-assisted entanglement concentration of partially entangled multiphoton W states with linear optics. *J. Opt. Soc. Am. B* **29**, 1685 (2012).
- [47] Du F.F., Li T., Ren B.C., Wei H.R., Deng F.G.: Single-photon-assisted entanglement concentration of a multiphoton system in a partially entangled W state with weak cross-Kerr nonlinearity. *J. Opt. Soc. Am. B* **29**, 1399 (2012).
- [48] Zhou L., Sheng Y.B., Cheng W.W., Gong L.Y., Zhao S.M.: Efficient entanglement concentration for arbitrary less-entangled NOON states. *Quant. Inf. Process.* DOI 10.1007/s11128-012-0472-x (2012)
- [49] Nemoto K., Munro W.J.: Nearly deterministic linear optical controlled-nOT gate. *Phys. Rev. Lett.* **93**, 250502 (2004) .
- [50] Lin Q., Li J.: Quantum control gates with weak cross-Kerr nonlinearity. *Phys. Rev. A* **79** 022301 (2009).
- [51] Barrett S.D., Kok P., Nemoto K., Beausoleil R.G., Munro W.J., Spiller T.P.: Symmetry analyzer for nondestructive Bell-state detection using weak nonlinearities. *Phys. Rev. A* **71** 060302(R) (2005).
- [52] Xia Y., Chen Q.Q., Song J., Song H.S.: Efficient hyperentangled Greenberger-Horne-Zeilinger states analysis with cross-Kerr nonlinearity. *J. Opt. Soc. Am. B* **29**, 1029 (2012).
- [53] Lin Q., He B.: Single-photon logic gates using minimal resources. *Phys. Rev. A* **80**, 042310 (2009).
- [54] Lin Q., He B.: Efficient generation of universal two-dimensional cluster states with hybrid systems. *Phys. Rev. A* **82**, 022331 (2010).
- [55] He B., Ren Y., Bergou J.A.: Creation of high-quality long-distance entanglement with flexible resources. *Phys. Rev. A* **79**, 052323 (2009).
- [56] He B., Ren Y., Bergou J.A.: Universal entangler with photon pairs in arbitrary states. *J. Phys. B* **43**, 025502 (2010).
- [57] He B., Lin Q., Simon C.: Cross-Kerr nonlinearity between continuous-mode coherent states and single photons. *Phys. Rev. A* **83**, 053826 (2011).
- [58] Munro W.J., Nemoto K., Beausoleil R.G., Spiller T.P.: High-efficiency quantum-nondemolition single-photon-number-resolving detector. *Phys. Rev. A*, **71**, 033819 (2005).
- [59] Duan L.M., Lukin M.D., Cirac J.I., Zoller P.: Long-distance quantum communication with atomic ensembles and linear optics. *Nature*, **414**, 413 (2001).
- [60] Osorio C.I., Bruno N., Sangouard N., Zbinden H., Gisin N., Thew R.T.: Heralded photon amplification for quantum communication. *Phys. Rev. A* **86** 023815 (2012).